Abstract:
This paper suggests a method for designing PSS to damp multi-machine power system oscillations. The method is based on robust control theory. First, the conventional method for designing robust controller in LMI framework is illustrated. Then, the suggested method is given, in which, a PID output feedback controller is tuned using the LMI approach. Mostly, the classical methods are used to tune a PSS, but in this paper a robust approach is investigated to guarantee the robustness of the given PSS. The performance of the controller is tested on a sample power system. Simulation results show the effectiveness, robustness, and good performance of the suggested controller.

Keywords: Decentralized control, Linear matrix inequalities, Large-scale system, Power system stabilizer (PSS), Robust control.

1. Introduction
Power system stabilizers (PSS) have been used for many years to add damping to electromechanical oscillations. They use auxiliary stabilizing signals to control the excitation system so as to improve power system dynamic performance. Commonly used input signals to the power system stabilizer are shaft speed and accelerating power. The power system dynamic performance is improved by damping system oscillations. This is a very effective method of enhancing small-signal stability performance [1].

In order to damp power system oscillations, most authors use the classical PSS, which is not robust due to uncertainties. Some others use methods such as pole placement [2], adaptive control [3], robust methods [4] and etc, for designing PSS. These methods lead to a high order PSS which is not applicable in practice. Sometimes, it is needed to reduce the order of the controller to make it more useful for practical purposes [4]-[7], but the order reduction method decreases the controller performance in some aspects.

Here, two methods for robust PSS design are investigated. First, the conventional robust approach is analyzed and the advantages and the disadvantages of the method are discussed. Then, the suggested approach is presented. Both approaches are based on robust control theory.

The conventional approach is based on $H_{\infty}$ theory and formulates the problem in the LMI (Linear Matrix Inequalities) framework. It uses the LMI toolbox in MATLAB [8], to solve the optimization problem.

In [9]-[13] different approaches for robust controller design in power systems are suggested. All of these approaches result in high-order dynamic PSSs which are very difficult to be implemented in practice.

In this paper, the suggested approach is based on robust control theory and results in a simple and practical controller. A PID controller with unknown coefficients is considered to be installed on each subsystem in a large-scale power system. The method uses the LMI approach to find the optimal controller coefficients. This method guarantees the robustness of the PSS and results in an output-feedback controller which is so simple for implementing in practice.

The performance of the controller is tested on a sample power system. Simulation results show the effectiveness, robustness, and good performance of the suggested controller.

The paper is organized as follows: The system model is described in Section II. The conventional robust control design method based on LMI approach and the suggested approach of tuning robust PID controller are presented in...
section III, parts A and B, respectively. The method is applied to a simulated model of a sample power system in section IV. Finally, the conclusion is given in section V.

2. Large-scale Power System Model

A multi-machine power system is a large-scale nonlinear system. To evaluate the dynamic response of a power system of \( n \) generators and \( m \) total buses, the following equations (the synchronous generator d-q axis model and the exciter simple model) should be written for each generator \([14]\). (14)

\[
\frac{d\delta_i}{dt} = \omega - \omega_i
\]

\[
M_i \frac{d\omega_i}{dt} = -E'_{qi} - (X'_{dq} - X'_{dq}) I_{di} + E_{fdi}
\]

\[
T_{dqi} \frac{dE_{di}'}{dt} = -E'_{di} - (X'_{dq} - X'_{dq}) I_{di} + E_{fdi}
\]

\[
\frac{dE_{fdi}}{dt} = T_{refi}^{-1} \left[ K_{refi} (V_{refi} - V_i - E_{fdi}) - K_{el} E_{fdi} \right]
\]

For all generators, in addition to the dynamic equations, two sets of algebraic equations are appointed.

\[
v_i \cos \theta_i + B_i \left( I_{di} \sin \delta_i + I_{qi} \cos \delta_i \right) - X'_{di} \left( I_{di} \sin \delta_i - I_{qi} \cos \delta_i \right) = 0
\]

\[
v_i \sin \theta_i + B_i \left( I_{di} \sin \delta_i - I_{qi} \cos \delta_i \right) + X'_{qi} \left( I_{di} \sin \delta_i + I_{qi} \cos \delta_i \right) = 0
\]

\[
P_i = V_i \left( I_{di} \cos (\delta_i - \theta_i) + I_{qi} \sin (\delta_i - \theta_i) \right) + P_{refi}(V_i)
\]

\[
Q_i = V_i \left( I_{di} \sin (\delta_i - \theta_i) + I_{qi} \cos (\delta_i - \theta_i) \right) + Q_{refi}(V_i)
\]

Finally, the net active and reactive powers at each of the network buses in terms of the voltage magnitude, conductances and admitances are given by:

\[
P_i = \sum_{h=1}^{n} V_i V_h (G_{hij} \cos \theta_{ih} + B_{hij} \sin \theta_{ih})
\]

\[
Q_i = \sum_{h=1}^{n} V_i V_h (G_{hij} \sin \theta_{ih} - B_{hij} \cos \theta_{ih})
\]

Some information about the parameters definition is given in the appendix. The details of this model are given in \([14]\). The large-scale power system is consisted of different subsystems, the dynamic model of each, is given by (1)-(5). In power systems, the power system stabilizers (PSS) are used to add damping to electromechanical oscillations. The generator and PSS block diagrams and their interconnections are shown in Fig. 1. A PSS block receives its input from the generator block and transmits the change of reference voltage, which is the input to generator block. Here, the generator rotor speed is considered as the input to the PSS.

\[
\frac{dAx}{dt} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} Ax \\ Ay \end{bmatrix} + \begin{bmatrix} B_a \\ 0 \end{bmatrix} A \Delta u
\]

Where \( \Delta x \) and \( \Delta y \) are state variations and algebraic parameter variations in the neighborhood of the operating point, respectively. \( \Delta u \) (system input) is the variation of reference voltage (\( \Delta V_{ref} \)) in (5).

Eliminating \( \Delta y \) in (12), one has:

\[
\frac{dAx}{dt} = \left( A - BD^{-1}C \right) Ax + B_a \Delta u
\]

Equation (13) is the dynamic model of the whole system. The matrix \( \tilde{A} \) for an \( n \) machine system is of the following form:

\[
\tilde{A} = \begin{bmatrix} A_1 & G_{12} & \cdots & G_{1n} \\ G_{21} & A_2 & \cdots & G_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ G_{n1} & G_{n2} & \cdots & A_n \end{bmatrix}
\]

Where, \( G_{ij} \) is the interaction between subsystems i and j (generators i and j). Specifically, the dynamic model for subsystem 1 is:
\[ \frac{d \Delta x_i}{dt} = A_i \Delta x_i + G_{i2} \Delta x_2 + \cdots + G_{in} \Delta x_n + B_{iu} \Delta u \]  

(15)

The term \( G_{i2} \Delta x_2 \) is the interaction effect of subsystem 2 on subsystem 1. In this paper, all the terms of the form \( G_{ij} \Delta x_j \) are treated as disturbances and robust controllers are designed to cope with them.

It is obvious from the model (1)-(5) that the interaction terms are resulted from the direct and quadrature axis stator currents. Since the current of each generator is bounded [16], the interaction terms in each subsystem are bounded. Due to this important fact, the interaction terms can be treated as bounded disturbances. In section III, a robust controller is designed to overcome these disturbances.

Considering the interactions as disturbances, the details of system model “\( P_i(s) \)” in each subsystem ‘\( i \)’, are given in the following:

State equation:
\[ x_i(t) = A_i x_i(t) + B_i u_i(t) + B_i w_i(t) \quad i = 1 \cdots n \]
\[ z_i = C_{2i} x_i \]  

(16)

System states: \( x_i = [\Delta \delta_i \Delta \delta_q \Delta E_{qi} \Delta E_{di} \Delta E_{fdi}] \)

Input: \( u_i = \Delta V_{refi} \)

Disturbance: \( w_i = \text{interaction of other subsystems} \)

Measured output: \( y_i = \Delta \delta_i \)

Desired output: \( z_i = \Delta \delta_q \)

The \( A_i, B_j, \) and \( C_j \) matrixes can be computed using (1)-(5) and (12)-(13).

3. Robust Control

3.1. Robust Control via LMI Formulation [8, 17] - Conventional Approach

Robust control theory deals with minimizing the closed-loop RMS gain from \( w \) to \( z \) (Fig. 2). The signal \( z \) is a set of desired variables that needs to be controlled in the presence of a disturbance. The objective is to design a control law “\( u \)” based on measured variables “\( y \)” such that the effects of the disturbance \( w \) on the desired variables \( z \), as expressed by the norm of its transfer function \( \| P_{zw} \| \) is minimized.

Partition the plant \( P(s) \) as:
\[ \begin{bmatrix} Z(s) \\ Y(s) \end{bmatrix} = [T_{zw}(s) \quad T_{zw}(s)] [w(s) \quad U(s)] \]  

(17)

The closed-loop system transfer function will become:
\[ T_{zw} = T_{zw} + T_{zw} K(L - T_{yw})^{-1} T_{yw} \]  

(18)

The optimal \( H_\infty \) control seeks to minimize \( \| F_{zw} \|_\infty \) over all stabilizing LTI controllers \( K(s) \). Alternatively, the suboptimal control problem specifies some value \( \gamma > 0 \) such that the closed-loop system is internally stable and \( \| F_{zw} \|_\infty < \gamma \).

In the LMI approach, the plant is given in state-space form by:
\[ \begin{align*}
    x &= A x + B_w w + B_u u \\
    z &= C_x x + D_{1w} w + D_{1u} u \\
    y &= C_x x + D_{2w} w + D_{2u} u \\
\end{align*} \]  

(19)

\( x \) is the system states vector, \( z \) and \( y \) are the desired and the measured outputs, respectively. \( w \) and \( u \) are the disturbance and the system input, respectively. The objective is to design a controller in the following state space model, such that the effects of the disturbance \( w \) on the desired variables \( z \), as expressed by the infinity norm of its transfer function \( \| F_{zw} \|_\infty \), are bounded.

\[ \hat{\gamma} = A_K \hat{\gamma} + B_K y \\
    u = C_K \hat{\gamma} + D_K y \]  

(20)

Combining the system and the controller models, provided that \( (A, B_r) \) is stabilizable and \( (A, C_r) \) is detectable, the following closed-loop state-space model will be achieved:
\[ \begin{align*}
    \dot{x}_c &= A_c x_c + B_c w \\
    z &= C_c x_c + D_c w \\
\end{align*} \]  

(21)

The \( H_\infty \) performance is directly optimized by solving the following LMI problem:

\textbf{Lemma 1} [8, 17]: The closed loop RMS gain from \( w \) to \( z \) does not exceed \( \gamma \) if and only if there exists a symmetric positive definite matrix \( X_\gamma \) such that:
The order of the dynamic controller that will be obtained by this method is the size of the system and hence very large in general. To avoid this large-size controller, the following method is suggested.

3.2. Robust PID output controller design via LMI approach- Suggested Approach

In practice, the power system stabilizer is not usually a large dynamic controller. It is usually a PID controller. Here, the LMI approach is used to tune the PID parameters in each subsystem such that the following performance index is minimized:

$$\gamma = \min \| T_{zw} \|$$  \hspace{1cm} (23)

Consider a large-scale system consisted of \( n \) subsystems. Let the controller in each subsystem be a PID with unknown coefficients (Fig. 3).

$$u_i(s) = \left( k_1 + \frac{k_2}{s} + k_3s \right) y_i(s)$$  \hspace{1cm} (24)

The third inequality guaranties the \( A \) matrix to be negative definite. This assures the stability of the system. Looking deeply to the second inequality, it is observed that there are the combination of \( P \) and \( \{ k_i \} \) parameters, so the above problem is not in a suitable format for linear matrix inequality approach. To solve this problem, one solution would be the ILMI (Iterative LMI) approach [19]-[20] which divides the problem to two simpler optimization problems, each being linear in the decision variables and then solves the problem iteratively. Actually, this approach changes the problem from optimal to a suboptimal one.

4. Simulation Results

The one-line diagram of the system under study is shown in Fig. 4. The system consists of two subsystems, which are separated by a dotted line. System parameters are tabulated in the appendix.
This diagram belongs to the southern part of Iran power system. The buses 4 and 6 are the points of attachment of this network to other parts of Iran grid. These two points are modeled by coherency method and the parameters of model are obtained by trial and error [14].

\[ G(s) = \frac{-10(1.04s^4 + 1.75)}{0.001s^5 + 0.055s^4 + 0.055s^2 + 1.9s + 1.3s + 1.4} \]  \hspace{1cm} (28)

First, the conventional LMI method which was discussed in section III part A is applied on the above transfer function. The designed dynamic controller of order 5 for the first subsystem in the state-space form of (20) is:

\[
 A = \begin{bmatrix}
 0.0317 & -0.0183 & 0.067 & 0.007 & 0.0012 \\
 1.9075 & -1.1814 & 2.1289 & 0.0143 & -0.0037 \\
 -4.877 & 0.3012 & -0.545 & -0.0078 & 0.0001 \\
 4.305 & -2.688 & 0.4786 & 0.0114 & 0.0008 \\
 -6.147 & 0.3934 & -0.6801 & -0.0502 & -0.0079 \\
\end{bmatrix}
\]

\[ B = 10^8 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ C = \begin{bmatrix} 0.0457 & 0.276 & 0.972 & -0.601 & -0.014 \end{bmatrix} \]

Note that due to the order of the system, which is 5, the obtained $H_\infty$ controller is of order 5, too. By modeling details, the order of the controller will become higher and equals the order of the plant. However, the PID controller always has order two. Using a more complicated plant model, which has a higher order, results in new PID parameters ($k_i$).

To design the PID output controller (the suggested method of this paper), the canonical controllable form is obtained, which is given in the following and the $A$ matrix is given on the top of the next page.

\[
 A = \begin{bmatrix}
 0 & 1 & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 & 0 \\
 0 & 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 0 & 1 \\
 17.5\times10^4k_2 & (-1.4+1039k_2+17.5k_5)\times10^4 & (-1.3+1039k_2+17.5k_5)\times10^4 & (-2+1039k_3)\times10^4 & -0.5 & -0.005 \\
\end{bmatrix}
\]

Using lemma 3, the algorithm is summarized in the following steps:

1-Select three $k_i$ parameters such that the inequality $A < 0$ is satisfied.

2-Keeping $k_i$ constant, minimize $trace(R)$ with respect to $P$ and $R$ in (27) and find $R$ and $P$.

3- Keeping $P_{ij}$ constant, minimize $trace(R)$ with respect to $k_j$, $P_{ij}$ \(i = 1 \ldots 5\) and $R$ in (27).

4-If the chosen stopping criterion is verified, stop else go to step 2.
To test the performance of the controllers, a disturbance is applied to the system. The disturbance is a change in the field voltage of generator number 1. In Fig. 5 the rotor angle deviation of the first subsystem (Generator no. 1), without PSS is shown. It is obvious that the system needs a long time to damp the oscillations.

The designed robust controller based on LMI framework, formulated in section III part A, is applied on the system and its simulation result is shown in Fig. 6.

To achieve an applicable controller which is robust under perturbations, the LMI toolbox is used to design PID controllers (The suggested approach in this paper-section III part B). The result is shown in Fig. 7.

As it is seen from the simulations, the rotor angle of the system without PSS has many oscillations to reach the steady state response (Fig. 5). In the other two Figures (Fig. 6-7), the rotor angle oscillations are damped so faster than the system without PSS.

The electrical power deviations of the system without PSS and the PID PSS, are shown in Fig 8 and Fig 9, respectively. In the system without controller, the electrical power has too much oscillations and it takes a long time for oscillations to be damped. It is obvious that using the PID controller, the oscillations of electrical power deviation are damped so fast and these brief oscillations are acceptable in practice.

It is seen from simulations that the rotor angle and the electrical power of the system without PSS has many oscillations to reach the steady state response, but the oscillations are damped so fast when a controller is applied on the system. When the high order conventional robust controller or PID controller is applied to the system, even in the first cycle, the peak of oscillation is limited.

Comparing Fig. 6 and Fig. 7, it is seen that the response of the system with $H_\infty$ controller is a bit better than the system with PID controller. The difference between these responses is so negligible. On the other hand, the PID controller cost less than the $H_\infty$ one. So, it is not
5. Conclusion

This paper investigates two methods of designing robust PSSs for damping multi-machine power system oscillations. The first one is the robust conventional approach which is formulated in LMI frame. The second one, which is the suggested approach of this paper, is an output feedback PID controller that is tuned using the LMI approach.

The performance of both controllers is tested on a sample power system model. The sample model has two subsystems. The simulation results for one of the subsystems are shown. Simulation results show that both controllers are effective, robust and have good performance. The first controller has smaller oscillations than the second one, but as it is a high-order dynamic controller, its implementation is difficult. The second controller (PID) is so easy to be implemented in practice, so it is advised to be used for practical purposes.
References